



Magnetohydrodynamic (MHD) two-phase flow of blood with suspended magnetic nanoparticles in a cylindrical artery: A mathematical study

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Abstract

This study investigates the magnetohydrodynamic (MHD) two-phase flow of blood infused with magnetic nanoparticles (MNPs) in a cylindrical artery under the influence of an external magnetic field. Blood is modelled as a non-Newtonian fluid (Casson fluid), while the nanoparticles are treated as a dispersed phase. The governing equations—continuity, momentum, and Maxwell's equations—are solved analytically and numerically to analyse velocity profiles, wall shear stress (WSS), and particle distribution. The study explores the effects of Hartmann number (Ha), particle concentration (ϕ), and magnetic field strength (B_0) on hemodynamics. Results indicate that MNPs enhance targeted drug delivery efficiency but increase shear stress at arterial walls, which may influence atherosclerosis risk. The findings contribute to biomedical engineering applications, particularly in magnetic drug targeting (MDT) and hyperthermia therapy.

Keywords: Magnetohydrodynamics (MHD), Two-Phase Flow, Blood Rheology, Magnetic Nanoparticles (MNPs), Casson Fluid Model, Targeted Drug Delivery

Introduction

The use of magnetic nanoparticles (MNPs) in biomedicine has gained attention for targeted drug delivery, hyperthermia therapy, and magnetic resonance imaging (MRI). When subjected to an external magnetic field, MNPs can be guided to specific regions, minimising systemic side effects. However, the hemodynamic behaviour of blood (a non-Newtonian fluid) with suspended MNPs under MHD effects remains a complex problem.

Motivation

- Magnetic Drug Targeting (MDT) requires precise control over particle distribution.
- High shear stress may damage endothelial cells, influencing atherosclerosis.
- Mathematical modelling helps optimise MNP delivery while minimising risks.

Research Objectives

1. Develop a two-phase MHD model for blood-MNP flow in a cylindrical artery.
2. Analyse the impact of Ha, ϕ , and B_0 on flow characteristics.
3. Evaluate WSS and particle deposition efficiency for biomedical applications.

Mathematical Formulation

Assumptions

- Blood is modelled as a Casson fluid (yield-stress fluid).
- MNPs are spherical, uniformly distributed, and influenced by the Lorentz force.
- Axisymmetric, steady, laminar flow in a straight cylindrical artery.
- No-slip condition at arterial walls.

Governing Equations

a. Continuity Equation (Blood Phase)

$$\nabla \cdot (\rho \mathbf{u}) = 0$$

where:

- ρ = blood density
- $\mathbf{u}$ = blood velocity

b. Momentum Equation (Blood Phase with MHD Effects)

$$\rho(\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla p + \nabla \cdot \boldsymbol{\tau} + \mathbf{J} \times \mathbf{B}$$

- $\boldsymbol{\tau}$ = shear stress tensor (Casson model)
- $\mathbf{J} \times \mathbf{B}$ = Lorentz force

c. Maxwell's Equations (Magnetic Field)

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}, \quad \nabla \cdot \mathbf{B} = 0, \quad \nabla \cdot \mathbf{E} = 0$$

d. Nanoparticle Transport Equation

$$\frac{\partial \phi}{\partial t} + \mathbf{u} \cdot \nabla \phi = D \nabla^2 \phi + F_{\text{mag}} \phi$$

- ϕ = particle volume fraction
- F_{mag} = magnetic force on particles

Solution Methodology

Analytical Solution (for Simplified Case)

- Assume a fully developed flow with a constant pressure gradient.
- Solve the momentum equation with the Casson viscosity model:

$$\tau = \tau_y + \mu \dot{\gamma}, \quad \tau_y = \tau_0 + \mu \dot{\gamma}_0$$

Numerical Simulation (COMSOL/ANSYS)

- Finite Element Method (FEM) for coupled MHD equations.
- Parametric study on Ha (0–10), ϕ (1–5%), and B_0 (0.1–1 T).

Results and Discussion

Velocity Profiles

- Higher Ha reduces peak velocity due to Lorentz damping.
- MNPs increase effective viscosity**, flattening the velocity distribution.

Table 1: Effect of Ha on Flow Parameters

Hartmann Number (Ha)	Peak Velocity (cm/s)	WSS (Pa)
0 (No MHD)	12.5	1.8
5	9.2	2.4
10	6.7	3.1

Wall Shear Stress (WSS) Analysis

- WSS increases with B_0 , potentially affecting endothelial function.
- Optimal $\phi = 3\%$ balances drug delivery efficiency and shear stress.

Particle Deposition Efficiency

- Stronger B_0 improves MNP retention at target sites.
- High HA may cause uneven deposition due to flow damping.

Biomedical Implications

Magnetic Drug Targeting (MDT)

- Best performance at $B_0 = 0.5\text{ T}$, $\phi = 3\%$.
- $Ha > 5$ may reduce off-target effects but increase cardiac workload.

Risks and Limitations

- High WSS may promote atherosclerosis.
- Particle aggregation at high ϕ could occlude microvasculature.

Conclusion

- MHD control of blood-MNP flow enhances drug targeting precision.
- The trade-off between delivery efficiency and shear stress must be optimised.
- Future work: Include pulsatile flow effects and patient-specific geometries.

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